

A Letter from Mr. John Collins to the Reverend and Learned Dr. John Wallis Savilian Professor of Geometry in the Univerfity of Oxford, giving his thoughts about some Defects in Algebra.

TO describe the *Locus* of a cubick \mathcal{A} Equation.

A Cardanick \mathcal{A} Equation convenient for the purpose, (*viz.* such as shall have the dioriftick limits rational) must have the Coefficient of the roots to be the triple of a square

number such is $a^3 - 48a = N$.

Assume a rank of roots in Arithmetical progression, and raise resolvends thereto $a^3 - 48a = N$ or resolvends.

R	N
Such are	1 1-----48=47
	2 8- ---96=88
	3 27---144=17
	4 64 --192=128
	5 125--240=115
	6 216- -288=72
	7 343- -336=+7
	8 512-384=+128
	9 729-432=+297

Draw a *Base* line and a perpendicular thereto, and from O in the *Base* line prick the negative *resolvends* downwards, and the affirmative ones upwards, and raise their roots upon them as ordinates, a *Curve* passing through the same is one Moity of the *Curve* or *Locus* on the right hand for affirmative roots and the other moity on the left hand is described in the same manner by assuming a rank of negative roots, and raising resolvends thereunto. The *Curve* Fig. 4. may give a resemblance of the thing.

And 16 the third part of the Coefficient of the roots cubed is equal to the square of 64 half the *resolvend*, or *dioriftick* limit.

Which in composing of *Cardans* canon is always substracted from the square of half the *absolute*, as in the example following.

If I were to find the root belonging to the *resolvend* 297

The square of half thereof is $\frac{22052\frac{1}{4}}{4}$

The square of 64 half the dioriftick Limit $\frac{496}{4}$

The difference is $17956\frac{1}{4}$

And the rule is $148\frac{1}{2} \pm \sqrt{17956\frac{1}{4}}$

$148\frac{1}{2} \pm \sqrt{17956\frac{1}{4}}$

That is in a quadratick \mathcal{A} Equation, if 297 were the sum of the two roots and 64 the root of the *Rectangle*: then if from the square of half the sum, the rectangle be subducted, there remains the square of half the difference of the

Z
roots,



roots, and giving them an universal *Cube* root, it is.

$\sqrt[3]{148\frac{1}{2}} + \sqrt[3]{17956\frac{1}{4}} + \sqrt[3]{148\frac{1}{2}} - \sqrt[3]{17856\frac{1}{9}}$ to 9 the root sought.

In the former Scheme Q , B . and Q P . may signifie the roots of *Cardans Binomials* that run infinitely upward, and terminate at Q , as is mentioned in *Section the 5th*. And if they can be continued downwards, probably they will terminate at O . and R . The *touch* line in *Section 2d*. may here be represented by the line 9 S . and the *Chord* line between 9 and 8 by T . from whence tis plain that any root between 9 and 8 found near, may be limited by Approximations of *Majus* and *Minus*.

As to *CARDANS RULES*

1 The description of the *Locus* is before handled.

2 The *touch* line affording approaches by an *Æquation* derived out of that proposed is before described, and the method of drawing is mentioned by *Dr. Wallis* in the *Transactions*.

3. The Limits are of two kinds (*viz.*) either the *Base* limits when the resolvend is O . and the *æquation* falls a degree lower: or the *dioristick* limits whereby a pair of roots gain or loose their possibility, as is before described.

4 *Cardans* canons are but the sum of the roots of a solid quadratick *æquation* arising out of half the *dioristick* limit as the v of the rectangle, and the resolvend as the summ

5 If the roots of those *binomials* are separately prickt down as ordinates on their *resolvends*, they beget *curves* infinitely continued upward, and meeting in a point bisecting the root that is equal to a pair of equal roots, when the *æquation* is just limited, or *dioristick* as aforesaid in the Figure at Q .

6 If these *binomials* are prickt down as ordinates to their *resolvends*, *Mr. Newton* upon sudden thoughts, supposed they may describe both sides of an *Hyperbole*.

7 If so they cannot be continued downwards, but by the method in *Mercators Logarithmotechnia*: most numbers of a constant habitude belonging to any arithmetical progression, may by aid of the differences, and a Table of Figurative numbers (yea, and I add otherwise) be continued upward or downward, and if these run downward they will probably end both in the *base* limits at O and R .

8 If these binomial *curves* be continued downward, and separately found should always added make the root of a cubick *Æquation* capable of 3 roots: then *Cardans* impossible or negative roots are prov'd possible, and we only in ignorance how to extract them.

9 Assume any root within the limits of 3 possible roots, and raise a resolvend to it, and when you have done, by *Cardan's Rules* improved; you may find that root, and, with a little varyng

rying the same, both the other roots (as in the Postscript): for every number or magnitude capable of a *cube* root, is capable of two more, see *Section the 11th*. following.

10 If the roots in the former Section, be assumed in Arithmetical progression, and the æquation with its several Resolvends be depressed, there will come out a regular Series of Quadratick Æquations, whence an easie method will rise of writing down such ranks as multiplied by an Arithmetical progression, shall always beget the same cubic æquation, the Resolvend only varying.

11 Let the roots of this series of quadraticks be found as usual in binomials, let these binomials be cubed, and then let it be observed, whether the results are constant portions of the square of the Resolvend and of the dioristick limit: and if so, *Cardans* Rules will have their defect supplied.

12 In breaking a biquadratick, 'tis asserted that by leaving the Resolvend at liberty, it may be infinitely and rationally done, without the Aid of the separating cubic Æquation.

13 But supposing such separating cubic in store, of which *Bartholinus* in his dioristick hath given us great furniture in *Species*, why may not several roots of that æquation be assumed rational, and thence the biquadratick broken into as many pairs of quadratick æquations?

14 May not from hence a method arise of writing down 2 Series of quadraticks that multiplied together shall always beget the same biquadratick Nomes, the Resolvend only varying? and hence the *Locus* of the æquation is easily described.

15 Here again (as in the 11) if the binomial roots of these quadraticks be squaredly squared, and those results are constant portions of the cube of the Resolvend, and the dioristick limit; it will be certain there may be general surd Canons for æquations of the 4th. dimension, and *Monsieur Cluverius* (now at *London*) positively asserts he hath a general method to obtain them for all Dimensions.

16 As *Cardans* are surd canons deriv'd from the Resolvend, and dioristick limit, so it were worthy disquisition, whether other surd Canons (of which many are fitted to particular cases by your self, *Leibnitz* and others) do not arise out of the limits of those particular cases and æquations, and whether the glimpse of a general Method might thence be deriv'd for all other æquations, though encumbered with negative quantities? which Mr. *Gregory*, a little before his death, said he had attained.

17 The Learned Dr. *Pell* hath often asserted that after the Limits of an æquation are once obtain'd, the

fy to find all the roots to any Resolvend offer'd.

Now for instance (according to *Huddens* method) in a biquadratick æquation, you must multiply all the terms beginning with the highest, and so in order by 4, 3, 2, 1. and the last term or Resolvend by 0. whereby it is destroyed, and you come to a cubick Æquation, the same as *Farrriot* uses to take away the penultima Term of the biquadratick, the roots whereof being found, and as roots having Resolvends raised thereto in the biquadratick Æquation, are the dioristick Limits thereof.

18 And if this easy method were known, we may come down the Ladder to the bottom, and fall into irrational quantities, and ascend again. Against which assymetry, an Æquation might be assumed low, as a rational quadratick, and thence a cubick Æquation formed, whose limits should be found by aid of the quadratic Æquation, and out of that cubick a Biquadratick Æquation, whose limits should be found by the aid of that cubick Æquation, &c.

19 Æquations may be so continued of two Nomes, that both the dioristick and base limits, should be rational, then supposing such Æquation incomplete, the increasing or diminishing the roots, fills up all the vacant places.

Q. Whether or in what place one or both sorts of Limits shall loose their rationality? And what is the nature of the roots thus drawn? in this I think you have already determined in divers of your sord Canons.

20 What Dr. *Pells* method mention'd in *Section* 17 should be I cannot guess, unless it be either.

To make sord Canons. Or good approaches.

Or that raising Resolvends out of assumed roots, those should make a store from whence to derive the roots of the Resolvend offered.

Or making quadratick Æquations out of the dioristick and base limits, those might be interpolated, by aid of a Table of figurate numbers, or otherwise thereby, as in quadratick Æquations to attain two roots of a biquadratick at once. which if performed the greatest difficulties are overcome, and why should not this seem probable, in regard the *Curve* or *Locus*, be the Æquation what it will, makes indented porches.

21 Suppose I should propound two cubick or biquadratick Æquations, in both whereof all the signs are +. It is propounded out of these two, to derive a third Æquation, whose root shall be the Summ, Difference, or Rectangle of the Roots of the two Æquations propounded. This Mr *Gregory* a little before his death writ word he had obtained and in the following Series for finding the Moity of a Hyperbolick Logarithm I suppose made use of.

From

From a number propos'd subtract an Unit, let that be Numerator, and to it add an Unit, let that be Denominator, and call that fraction N .

Then $N + \overset{1}{N} + \overset{3}{N} + \overset{5}{N} + \overset{7}{N} + \overset{9}{N} + \overset{11}{N} + \overset{13}{N}$, &c. is

Equal to half the Hyperbolick Logarithm sought.

EXAMPLE in the Number 2.

N

The Fraction is $\frac{1}{2}$	1,	333333	=	333333
	3,	370370	=	123456
	5,	41152	=	8230
The Rank N is easily	7,	4572	=	653
made by dividing ev'ry	9,	508	=	56
preceding number by	9.11,	56	=	5
	13,	6	=	0

3465733

6931466² which is

The Hyperbolick Logarithm of 2 sought.

I want time to consider the premises, but hope you will, (in regard you seem to think it strange that any difficulties should remain about Cubicks that are not presently resolved) your considerations wherein will be very acceptable and worthy publick view.

Other Series in Print of *Mercator*, &c. dispatch not as this doth neither thereby can the Logarithm of 2 be easily made, but by making the Logarithms of such mixt numbers or fractions that multiplied together make the result 2 just as $2 \times 1\frac{1}{2} = 3$; whence having and finding that of $\frac{1}{2}$, you presently have the Logarithm of 3.

2 A *Cardanick* Equation that is a Cubick one wanting the second term, may be multiplied or divided by a rank of continual proportionals, so as to render the coefficient of the roots canonick, that is, to make it the same with the Equations of the Table, that find the Sine, Tangent, or Secant of the third part of that arch to which any Sine, Tangent, or Secant is propounded, and so finding the roots in the tables, those sought are thence obtained by Multiplication or Division. Yea, and the coefficient of the roots may in like manner be rendred an Unit, and then the Resolvends sought in a table of the sums or differences of the Cubes of numbers and their roots, shall help you to such roots, as multiplied or divided as aforefaid shall be the true ones sought.

23 It is an enquiry worth consideration, whether two of the roots of a biquadratick may not be kept constant, and

the

the rest be increased or diminished, either Arithmetically, or by multiplication and division in a known *Ratio*? certainly regular Progressions will arise, though as yet, we cannot increase the true roots of an Æquation without as much diminishing the Negative nor can we multiply or divide the roots without we alter all of them, and consequently cannot reduce coefficients to such habitades as are desirable.

24 It is a pleasant concinnity out of a root to raise a Resolvend without raising any of the Powers of the root, and at the same time without a thorough binomial Division to depress the Æquation a degree lower.

EXAMPLE.

Let the Æquation be $a^4 + 10a^3 + 6a^2 + 20a = 1072$.

Let the root be 4, the resolvend is thus raised by adding the coefficients as you go, and multiplying by the root, thus $4 + 10 = 14 \times 4 = 56 + 6 = 62 \times 4 = 248 + 20 = 268 \times 4 = 1072$, with the same work the Æquation may be depressed without Division.

EXAMPLE.

Let the Æquation be as before, and place the root with the former products underneath respectively, the sum is the depressed Æquation.

$$\begin{array}{r} a^4 + 10a^3 + 6a^2 + 20a - 1072 = 0 \\ + 4 \\ 56 + 248 + 1072 \end{array}$$

The sum $a^4 + 14a^3 + 62a^2 + 268a = 0$. that is divided by a.

$a^3 + 14a^2 + 62a + 268 = 0$. which is the under Æquation sought found without Division

25 It's conceived that all Æquations may be so regulated as to be reduced to as many Arithmetical Progressions of multipliers in whole numbers, as the Æquation hath dimensions, where of one of the progressions shall be a Series of roots: hence the raising Resolvends by tentative work is rendred Logarithmetical For Example write down any 3 arithmetical Progressions, viz.

R	H	}	I say the Rank H are the Resolvends or <i>Homogenea Comparationis</i> of a cubick Æquation, whose roots are the Rank R. This cubick Æquation is easily attained out of the differences of the Rank R. for out of the Rank R. in any Æquation proposed raise separately the respective powers (with regard to their Coefficients) and out of the three ranks so raised compose their respective differences, and they shall be the same with the differences of the rank of Resolvends or <i>Homogenea Comparationis</i> here noted by H.
$1 \times 6 \times 3 = 18$			
$2 \times 7 \times 5 = 70$			
$3 \times 8 \times 7 = 168$			
$4 \times 9 \times 9 = 324$			

If

If such \mathcal{A} Equation be encombred with fractions they are all removed at once, by multiplying moſt conveniently, by the leaſt number that is diviſible by the Denominators of ſuch fractions, hence alſo the infinite Series before mentioned (and others) are reduceable to Logarithms.

26 Where \mathcal{A} Equations have all their terms adſected with the ſame ſign both $+$ or $-$) *Mr. Newton and Mr. Gregory* deſcended have affirmed they are all reduceable to ſome pure high power, which is of ſingular uſe in the infinite Series. And a Learned *German* where this cannot be done, hath aſſerted that they may be reduced to a higher power, with a variable Coefficient, which is the root ſought with a common addend or ſubtrahend. And even this would render an eaſy tentative Logarithmical way for attaining the root.

27 If but one Root of an \mathcal{A} Equation can be found at a time, then queſtionleſs a better Method is not yet attained, then what is mentioned in the printed propoſal about Printing *Mr. Bakers* Treatiſes therein mentioned.

28 Laſtly, as to Conſtructions for \mathcal{A} Equations, the following Probleme ſeems to be univerſal.

Any two analytick *Curves* (*viz.*) ſuch as wherein the Habitude between the Baſe and Ordinate may be expreſſed by an \mathcal{A} Equation being given in Magnitude and Poſition, and from the points of their interſection ordinates let fall to the *Axis* of either figure, or upon parallels to the ſaid *Axis*, the inquiry is of what \mathcal{A} Equation thoſe ordinates are the roots? *Dr. Barrow* liked the propoſition as well grounded, and left a diſcourſe about doing it in the conick Sections, in which there are 3 caſes, either the axes are parallel or being produced concur, beyond the vertexes of the figures without; or otherwiſe interſect within the figures. *Mr. Gregory* entred on the ſame contemplation, but death deprived us of the benefit of his thoughts.

Of Analytick (*alias* Geometrick) *Curves* there are innumerable forts, of which I ſhall mention one or two kinds.

Between an Arithmetical Progreſſion and its ſquares, or between its ſquares and its cubes, or its cubes and Biquadratics, there may be interpolated as many Arithmetical or Geometrical means as you pleaſe: and thence *Loci* or *Curves* deriv'd, which ſome call *Parabolics* or *Parabolasters*, ſee *Gregorius Geometricæ pars univerſalis* printed in *Italy* in Quarto.

Poſtſcript explaining Section the 9th.

After you have obtained the Cube roots of *Cardans* Binomials, according to *Van Schooten*, in *De Cas* or *Kerley*, if you change the Sines of the rational parts of thoſe roots, as alſo the

the Sines of the Radical Parts, and multiply those parts by 3, the results are also roots of the cubick Æquation first proposed.

EXAMPLE.

aaa --21a--2o____o
 The cube Roots of the Binomials are $+2\frac{1}{2} + \sqrt{-\frac{3}{4}}$
 $+2\frac{1}{2} - \sqrt{-\frac{3}{4}}$

Their sum is the Root sought $= +5$

And the other two Roots are $-\frac{5}{2} + \sqrt{2\frac{1}{4}}$
 $-\frac{5}{2} - \sqrt{2\frac{1}{4}}$

Also in this Æquation $a^3 - 6oa - 12 = 0$
 The Binomial Roots are $+4 + \sqrt{-4}$
 $+4 - \sqrt{-4}$

Hence the Root sought is $+8$
 And the other two roots are $-4 + \sqrt{+12}$
 $-4 - \sqrt{+12}$.

ADVERTISEMENT.

These papers were sent by Mr. Collins to Dr. Wallis in a Letter of 3 Oct. o. 1682, (with this Character, *I have sent you here with my thoughts about some defects in Algebra:*) and are a Copy of what he had written to some other (but I know not whom) to whom he speaks all along in the second person, whereas of others he speaks in the third person. And he did intend (had he lived longer) to perfect it further; by omitting some things which (though here he notes as defects) he found after to be done already, and supplying some others. But he lived not to perfect it, and therefore (that it be not lost) we here give it as we found it.

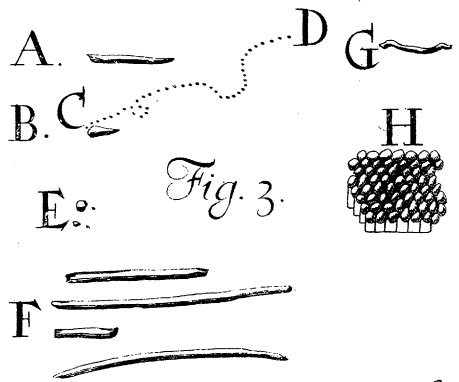
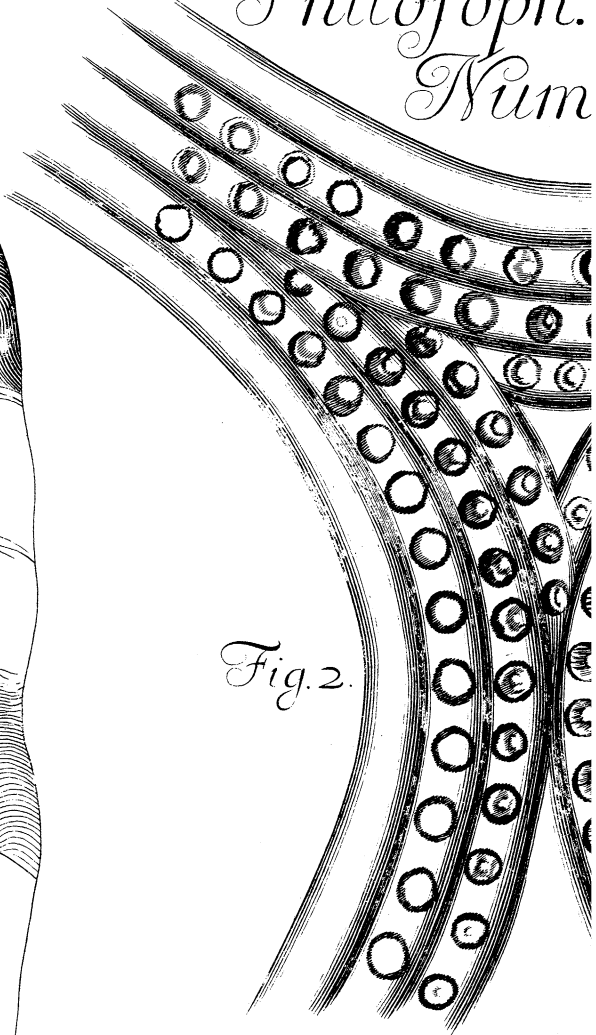
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Fig. 1.



Fig. 2.



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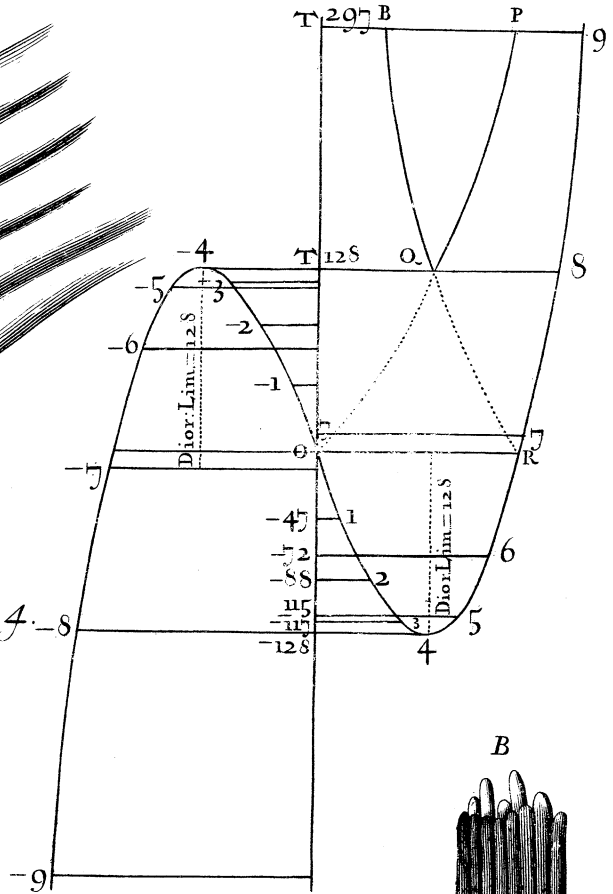
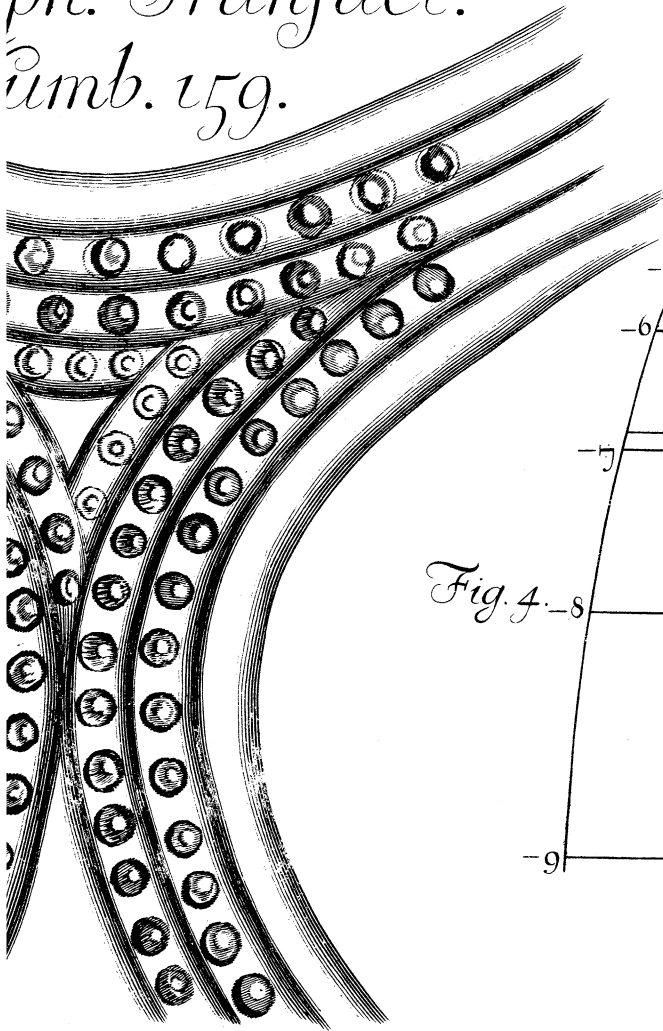
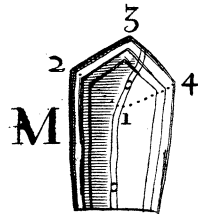
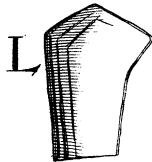
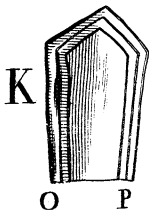
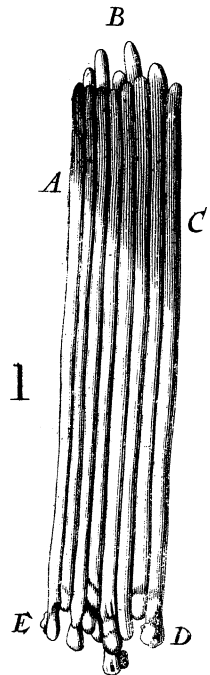


Fig. 4.



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Fig. 1.

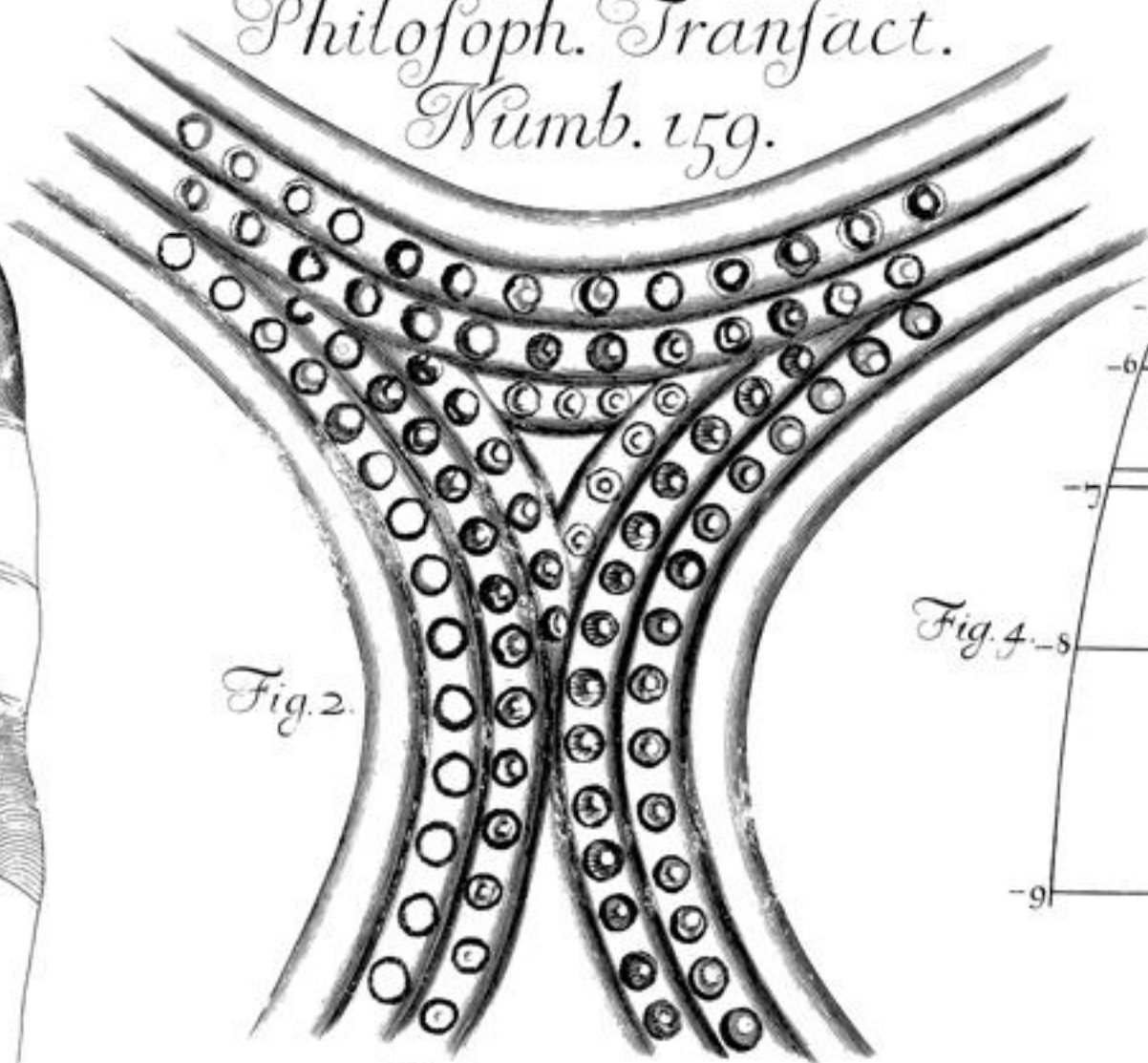


Fig. 2.

Fig. 4.

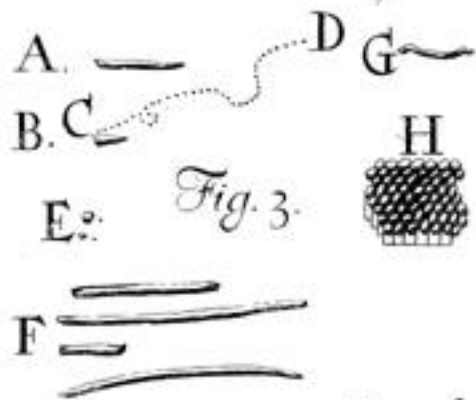
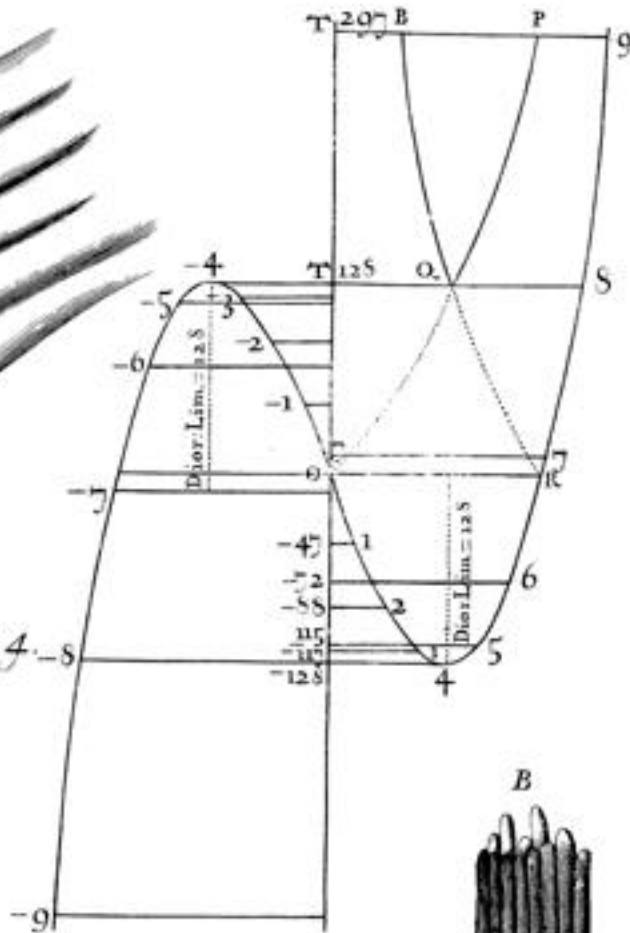


Fig. 3.

